Published by Institute of Physics Publishing for SISSA/ISAS
Received: *June 22, 2004*
Revised: *August 3, 2004*
Accepted: *August 9, 2004*

The effects of non-universal extra dimensions on the radiative lepton flavor decays $\mu \to e\gamma$ and $\tau \to \mu\gamma$ in the two Higgs doublet model.

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Abstract: We study the effect of non-universal extra dimensions on the branching ratios of the lepton flavor violating processes $\mu \to e\gamma$ and $\tau \to \mu\gamma$ in the general two Higgs doublet model. We observe that these effects are small for a single extra dimension, however, in the case of two extra dimensions there is a considerable enhancement in the additional contributions.

Keywords: Beyond Standard Model, Rare Decays.

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1. Introduction

Lepton flavor violating (LFV) interactions deserve to analyze since, theoretically, they are sensitive the physics beyond the standard model (SM) and they are rich in the sense that they exist at loop level and make it possible to predict the free parameters of the underlying theory.

The radiative LFV decays are among the well known candidates of the LFV processes and there are various experimental and theoretical works in the literature. The current limits for the branching ratios (BR's) of the $\mu \to e\gamma$ and $\tau \to \mu\gamma$ decays are 1.2×10^{-11} [1] and 1.1×10^{-6} [2], respectively. The extensive theoretical work has been done on these decays in the supersymmetric models [3], in the framework of the two Higgs doublet model (2HDM) [4, 5] and also in a model independent way [6].

In the general 2HDM, so called the model III, the possibility of the flavor changing neutral currents (FCNC's) at tree level allows the existence of the radiative LFV decays, theoretically. These decays are induced by the internal neutral Higgs bosons h^0 and A^0 and the Yukawa couplings carrying information about the strength of the lepton-lepton-neutral Higgs interactions are free parameters of the theory. These free parameters should be restricted by using the experimental measurements.

This work is devoted to the LFV processes $\mu \to e\gamma$ and $\tau \to \mu\gamma$ in the framework of the model III, with the inclusion of one (two) non-universal extra spatial dimension. The extra dimension scenario is based on the string theories as a possible solution to the hierarchy problem of the standard model (SM). The effects of extra dimensions on various phenomena have been studied in the literature [7]–[18]. For the extra dimension scenarios the starting point is a fundamental theory, lying in higher dimensions and the assumption that the four dimensional SM is its low energy effective theory. The procedure to pass from higher dimensions to the four dimension is the compactificitation of each extra dimension to a circle S^1 with radius R, which is a typical size of corresponding extra dimension. Finally, it leads to appear new particles, namely Kaluza-Klein (KK) modes of the particles in the theory. If all the fields feel the extra dimensions, so called universal extra dimensions (UED), the extra dimensional momentum is conserved, in other words, the KK number at each vertex is not violated. In the case of a single UED, the compactification size R has been predicted as large as few hundereds of GeV [7, 8, 10], in the range $200-500$ GeV, using electroweak precision measurements [11], the B-$\bar{B}$-mixing [12, 13] and the flavor changing process $b \to s\gamma$ [14]. If the extra dimensions are accessible to some fields but not all in the theory, they are called non-universal extra dimensions. In this case, the KK number at each vertex is not conserved and tree level interaction of KK modes with the ordinary particles can exist. In [15], it was observed that, a very light left handed neutrino can be obtained in the non-universal extra dimensions where only the right handed neutrino feels the extra dimension. The study in [16] is due to the effect of brane kinetic terms for bulk scalars, fermions and gauge fields in higher dimensional theories. There are also some phenomenological studies on the non-universal extra dimensions in the literature [17].

In the present work, we study the branching ratios (BR's) of the LFV processes $\mu \to e\gamma$ and $\tau \to \mu\gamma$ in the model III and we assume that the extra dimensions are felt by the new Higgs doublet and the gauge sector of the theory. The compactification of the single (double) extra dimension to the circle (torus) S^1 ($S^1 \times S^1$) brings new particles, KK modes of neutral Higgs bosons h^0 and A^0, and the additional vertices, inducing KK mode of neutral Higgs boson-lepton-lepton interaction, appear. In this case, the KK number in the vertices is not conserved, in contrast to the universal extra dimension (UED) case, where all fields experience the extra dimensions. The non-zero KK modes of neutral Higgs fields H have masses $\sqrt{m_H^2 + m_n^2}$ ($\sqrt{m_H^2 + m_n^2 + m_r^2}$) with $m_n = n/R$ ($m_n = n/R$, $m_r = r/R$). Here m_n ($m_n^2 + m_r^2$) is the mass (mass^2)of n'th (n, r'th) level KK particle for a single (double) non-universal extra dimension. Since the leptons have not KK modes and the additional effects due to the extra dimensions are the functions of the ratio $m^2_{\text{lepton}}/m^2_{KK\,\text{neutral Higgs}}$, there is a suppression in the numerical values of the expressions compared with the ones existing in the case of UED. However, we observe that, the BR's of the processes under consideration enhances considerably large for two non-universal extra dimensions, due to the crowd of neutral Higgs KK modes.

The paper is organized as follows: in section 2, we present the BR's of LFV interactions $\mu \to e\gamma$ and $\tau \to \mu\gamma$ in the model III version of the 2HDM with the inclusion of non-universal extra dimensions. Section 3 is devoted to discussion and our conclusions.

2. The effects of non-universal extra dimensions on the radiative lepton flavor decays $\mu \to e\gamma$ and $\tau \to \mu\gamma$ in the two Higgs doublet model

The radiative LFV decays exist at least in the one loop level and the theoretical values of their BR's enhance with the extension of the Higgs sector in the models beyond the SM. The model III version of the 2HDM accepts the flavor changing neutral currents (FCNC) at tree level and it is the candidate model to bring enhancement to the BR's of the radiative LFV decays. With the inclusion of possible non-universal extra dimensions which are felt by gauge bosons and the new Higgs particles, there appears an additional contribution to the BR of the decays under consideration. The part of lagrangian which carries the interaction responsible for the radiative LFV processes in 5 (6) dimension is

$$\mathcal{L}_Y = \xi^{E}_{5(6)\,ij} \bar{l}_i (\phi_2|_{y(z)=0}) E_j + \text{h.c.} \tag{2.1}$$

Here $\xi^{E}_{5(6)\,ij}$, with lepton family indices i, j, are 5(6)-dimensional dimensionful Yukawa couplings which can be rescaled to the ones in 4-dimension as $\xi^{E}_{5(6)\,ij} = \sqrt{2\pi R}\,(2\pi R)\,\xi^{E}_{ij}$, ϕ_i for $i = 1, 2$, are the two scalar doublets, l_i and E_j are lepton doublets and singlets, respectively. The scalar and lepton doublets are the functions of x^μ and y (y, z), where y (y, z) is the coordinate represents the 5(6)'th dimension. With the choice of the doublets ϕ_1 and ϕ_2

$$\phi_1 = \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2}\chi^+ \\ i\chi^0 \end{pmatrix} \right]; \qquad \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}H^+ \\ H_1 + iH_2 \end{pmatrix}, \tag{2.2}$$

and the vacuum expectation values

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}; \langle \phi_2 \rangle = 0\,. \tag{2.3}$$

it is possible to switch off the mixing between neutral scalar Higgs bosons and to separate the particle spectrum so that the SM particles are collected in the first doublet and the new particles in the second one.[1] Notice that the Yukawa matrices $\xi^{E}_{5(6)\,ij}$ are responsible for producing the FCNC at tree level and their entries are complex in general. Notice that, in the following, we replace ξ^E with ξ^E_N where "N" denotes the word "neutral".

In the case of the non-universal two extra dimensions where only the new Higgs field ϕ_2 is accessible to extra dimension in the Higgs sector, the compactification on a torus $S^1 \times S^1$, causes to appear KK modes $\phi_2^{(n,r)}$ of ϕ_2 in two spatial extra dimension as

$$\phi_2(x, y, z) \;=\; \frac{1}{(2\pi R)^{d/2}} \left\{ \phi_2^{(0,0)}(x) + 2^{d/2} \sum_{n,r}^{\infty} \phi_2^{(n,r)}(x) \cos\left(\frac{ny}{R} + \frac{rz}{R} \right) \right\} \tag{2.4}$$

where $d = 2$, the indices n and r are positive integers including zero but both are not zero at the same time. Here $\phi_2^{(0,0)}(x)$ the 4-dimensional Higgs doublet which includes the charged Higgs boson H^+, the neutral CP even (odd) h^0 (A^0) Higgs bosons. The KK mode of the charged Higgs boson has the mass $\sqrt{m^2_{H^\pm} + m_n^2 + m_r^2}$. Similarly, the neutral CP even (odd) Higgs h^0 (A^0) has the mass $\sqrt{m^2_{h^0} + m_n^2 + m_r^2}$ ($\sqrt{m^2_{A^0} + m_n^2 + m_r^2}$), where $m_n = n/R$ and $m_r = r/R$. Furthermore, we assume that the compactification radius R is the same for both dimensions. The expansion for a single extra dimension can be obtained by setting $d = 1$, taking $z = 0$, and dropping the summation over r. Notice that the gauge fields feel also the extra dimensions which results in KK modes after the compactification. However, they are irrelevant in our calculation since they do not play any role in the processes under consideration.

At this stage, we consider the flavor violating radiative decays of leptons, $\mu \to e\gamma$ and $\tau \to \mu\gamma$, by introducing a single non-universal extra spatial dimension. These decays exist

[1] Here H^1 (H^2) is the well known mass eigenstate h^0 (A^0).

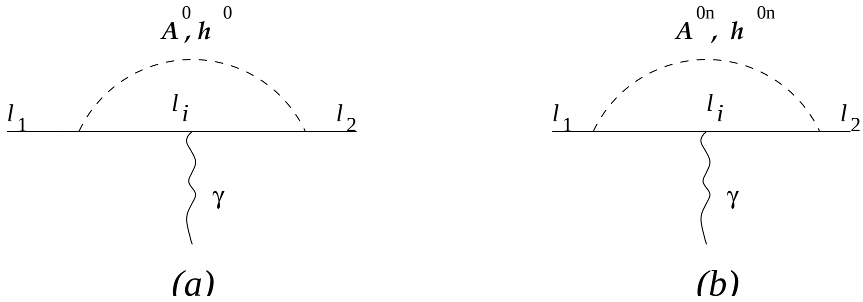

Figure 1: One loop diagrams contribute to $l_1 \to l_2\gamma$ decay due to the zero mode (KK mode) neutral Higgs bosons h^0 and A^0 (h^{0n} and A^{0n}) in the 2HDM.

at least at the one loop level with the help of the intermediate neutral Higgs bosons h^0 and A^0. With the inclusion of a single non-universal extra dimension, the KK modes of neutral Higgs bosons, namely h^{0n} and A^{0n}, contribute after the compactification (figure 1). Here the vertices involve two zero modes and one KK mode, since the KK mode conservation does not exist in the case of non-universal extra dimension, in contrary to the universal one.

In the loop calculations, the logarithmic divergences are eliminated by using the on-shell renormalization scheme. In this scheme, the self energy diagrams for on-shell leptons vanish since they can be written as

$$\sum(p) = (\hat{p} - m_{l_1}) \bar{\sum}(p) \, (\hat{p} - m_{l_2}) \, , \tag{2.5}$$

The divergences coming from the vertex diagrams can be eliminated by introducing a counter term V_μ^C so that we have the relation

$$V_\mu^{\mathrm{Ren}} = V_\mu^0 + V_\mu^C \, , \tag{2.6}$$

with the renormalized (bare) vertex V_μ^{Ren} (V_μ^0) and using the gauge invariance:

$$k^\mu V_\mu^{Ren} = 0 \, , \tag{2.7}$$

where k^μ is the photon four momentum vector.

Taking the vertex diagrams a and b in figure 1 and introducing only τ lepton for the internal line, the decay width Γ reads as

$$\Gamma(\mu \to e\gamma) = c_1(|A_1|^2 + |A_2|^2) \, , \tag{2.8}$$

where

$$\begin{aligned} A_1 = Q_\tau \frac{1}{48 m_\tau^2} \Bigg(& 6 m_\tau \bar{\xi}^{E*}_{N,\tau e} \bar{\xi}^{E*}_{N,\tau\mu} \Bigg((F(z_{h^0}) - F(z_{A^0})) + 2 \sum_{n=1}^{\infty} (F(z_{n,h^0}) - F(z_{n,A^0})) \Bigg) + \\ & + m_\mu \bar{\xi}^{E*}_{N,\tau e} \bar{\xi}^{E}_{N,\tau\mu} \Bigg(G(z_{h^0}) + G(z_{A^0}) + 2 \sum_{n=1}^{\infty} (G(z_{n,h^0}) + G(z_{n,A^0})) \Bigg) \Bigg) \, , \end{aligned}$$

$$A_2 = Q_\tau \frac{1}{48 m_\tau^2} \Bigg(6 m_\tau \bar{\xi}^D_{N,\tau e} \bar{\xi}^D_{N,\tau\mu} \Big((F(z_{h^0}) - F(z_{A^0})) + 2 \sum_{n=1}^{\infty} (F(z_{n,h^0}) - F(z_{n,A^0})) \Big) + $$
$$+ m_\mu \bar{\xi}^D_{N,\tau e} \bar{\xi}^{D*}_{N,\tau\mu} \Big(G(z_{h^0}) + G(z_{A^0}) + 2 \sum_{n=1}^{\infty} (G(z_{n,h^0}) + G(z_{n,A^0})) \Big) \Bigg) , \quad (2.9)$$

with $c_1 = \frac{G_F^2 \alpha_{em} m_\mu^3}{32\pi^4}$, left (right) chiral amplitudes A_1 (A_2). Similarly, the decay width of the LFV process $\tau \to \mu\gamma$ can be obtained as

$$\Gamma(\tau \to \mu\gamma) = c_2(|B_1|^2 + |B_2|^2) , \quad (2.10)$$

where

$$B_1 = Q_\tau \frac{1}{48 m_\tau^2} \Bigg(6 m_\tau \bar{\xi}^{E*}_{N,\tau\mu} \bar{\xi}^{E*}_{N,\tau\tau} \Big((F(z_{h^0}) - F(z_{A^0})) + 2 \sum_{n=1}^{\infty} (F(z_{n,h^0}) - F(z_{n,A^0})) \Big) + $$
$$+ m_\tau \bar{\xi}^{E*}_{N,\tau\mu} \bar{\xi}^{E}_{N,\tau\tau} \Big(G(z_{h^0}) + G(z_{A^0}) + 2 \sum_{n=1}^{\infty} (G(z_{n,h^0}) + G(z_{n,A^0})) \Big) \Bigg) ,$$
$$B_2 = Q_\tau \frac{1}{48 m_\tau^2} \Bigg(6 m_\tau \bar{\xi}^{D}_{N,\tau\mu} \bar{\xi}^{D}_{N,\tau\tau} \Big((F(z_{h^0}) - F(z_{A^0})) + 2 \sum_{n=1}^{\infty} (F(z_{n,h^0}) - F(z_{n,A^0})) \Big) + $$
$$+ m_\tau \bar{\xi}^{D}_{N,\tau\mu} \bar{\xi}^{D*}_{N,\tau\tau} \Big(G(z_{h^0}) + G(z_{A^0}) + 2 \sum_{n=1}^{\infty} (G(z_{n,h^0}) + G(z_{n,A^0})) \Big) \Bigg) , \quad (2.11)$$

with $c_1 = \frac{G_F^2 \alpha_{em} m_\mu^3}{32\pi^4}$, left (right) chiral amplitudes B_1 (B_2). The functions $F(w)$ and $G(w)$ in eqs. (2.9) and (2.11) read

$$F(w) = \frac{w\,(3 - 4\,w + w^2 + 2\,ln\,w)}{(-1+w)^3} ,$$
$$G(w) = \frac{w\,(2 + 3\,w - 6\,w^2 + w^3 + 6\,w\,ln\,w)}{(-1+w)^4} , \quad (2.12)$$

and $z_H = \frac{m_\tau^2}{m_H^2}$, $z_{n,H} = \frac{m_\tau^2}{m_H^2 + (n/R)^2}$, Q_τ is the charge of τ lepton, the dimensionfull Yukawa couplings $\bar{\xi}^E_{N,ij}$ appearing in the expressions are defined as $\xi^E_{N,ij} = \sqrt{\frac{4\,G_F}{\sqrt{2}}}\,\bar{\xi}^E_{N,ij}$. Furthermore, the theoretical results is due to the internal τ-lepton contribution since, we respect the Sher scenerio [19], results in the couplings $\bar{\xi}^E_{N,ij}$ $(i,j = e,\mu)$, are small compared to $\bar{\xi}^E_{N,\tau\, i}$ $(i = e,\mu,\tau)$, due to the possible proportionality of them to the masses of leptons under consideration in the vertices. Furthermore, we take the Yukawa couplings for the interactions lepton-lepton-KK mode of Higgs bosons (h^0 and A^0) as the same as the ones for the interactions of zero mode fields.

In the case two extra spatial dimensions which are felt by the second Higgs doublet in the Higgs sector, ϕ_2 can be expanded into its KK modes as in eq. (2.4) after the compactification of the extra dimensions on a torus $S^1 \times S^1$. In the two non-universal spatial extra dimensions the forms of the expressions (2.9) and (2.11) are the same, except that, the parameters $z_{n,H}$ and $\sqrt{m_H^2 + m_n^2}$ are replaced by , $z_{n,r,H} = \frac{m_\tau^2}{m_H^2 + (n/R)^2 + (r/R)^2}$ and $\sqrt{m_H^2 + m_n^2 + m_r^2}$. Furthermore, the number 2 in front of the summation should be replaced by 4 and the summation should be done for $n, r = 0, 1, 2 \ldots$ except $n = r = 0$.

Notice that, the additional KK mode contributions to the BRs of the processes we study are positive since

- the functions $F(w)$ and $G(w)$ (see eq. (2.12)), appearing in the expressions (2.9) and (2.11), are greater than zero and they increase with the increasing values of w, where w is always less than one;

- we have the condition $z_{n,h^0} > z_{n,A^0}$ $(n = 0, 1, 2, \ldots)$ coming from the assumption for the masses of neutral Higgs bosons h^0 and A^0, $m_{h^0} < m_{A^0}$.

3. Discussion

The radiative LFV decays $\mu \to e\gamma$ and $\tau \to \mu\gamma$ are controlled by the Yukawa couplings $\bar{\xi}^D_{N,ij}$, $i, j = e, \mu, \tau$ in the model III version of the 2HDM. These couplings should be restricted by using the experimental results, since they are free parameters of the theory. The dominant couplings are $\bar{\xi}^E_{N,\tau i}$, $(i = e, \mu)$ and $\bar{\xi}^E_{N,\tau i}$ $(i = \mu, \tau)$ for the $\mu \to e\gamma$ and $\tau \to \mu\gamma$ decays, respectively. Here, we assume that, the strength of these couplings are related with the masses of leptons denoted by the indices of them, similar to the Cheng-Sher scenerio [19]. This assumption forces us to consider only the τ lepton at the internal line for both processes. We also assume that the Yukawa couplings $\bar{\xi}^E_{N,ij}$ is symmetric with respect to the indices i and j. For the numerical values of these couplings we use the following restrictions:

- The upper limit of $\bar{\xi}^E_{N,\tau\mu}$ is predicted as $30\,\mathrm{GeV}$ (see [20] and references therein) by using the experimental uncertainty, 10^{-9}, in the measurement of the muon anomalous magnetic moment and assuming that the new physics effects can not exceed this uncertainty.

- For the Yukawa coupling $\bar{\xi}^E_{N,\tau e}$, we use broad range of the prediction, $10^{-3} - 10^{-2}\,\mathrm{GeV}$, which is obtained by using the experimental upper limit of BR of $\mu \to e\gamma$ decay, $BR \leq 1.2 \times 10^{-11}$ and predicted upper limit of $\bar{\xi}^E_{N,\tau\mu} \leq 30\,\mathrm{GeV}$ (see [4]).

- For the Yukawa couplings $\bar{\xi}^E_{N,\tau\tau}$, we use the numerical values which is greater than the upper limit of $\bar{\xi}^E_{N,\tau\mu}$.

With the addition of the extra dimensions, the new contributions to the physical parameters arise for these decays. In the case of non-universal extra dimensions, the KK number at each vertex is not conserved in contrast to the universal one case and lepton-lepton-KK neutral Higgs vertices appear. Due to the compactification of the a single (double) extra dimension on a circle (torus) S^1 $(S^1 \times S^1)$, the KK modes of internal neutral Higgs bosons h^0 and A^0 appear and they bring additional contributions to the physical quantities of the decays under consideration.

Our work is devoted to the prediction of the non-universal extra dimensions on the BR of the LFV processes $\mu \to e\gamma$ and $\tau \to \mu\gamma$, in the framework of the type-III 2HDM. We make this anaysis in one and two extra dimensions. In the case of two extra dimensions the crowd of KK modes cause to be more enhancement in the BR of the LFV decays compared

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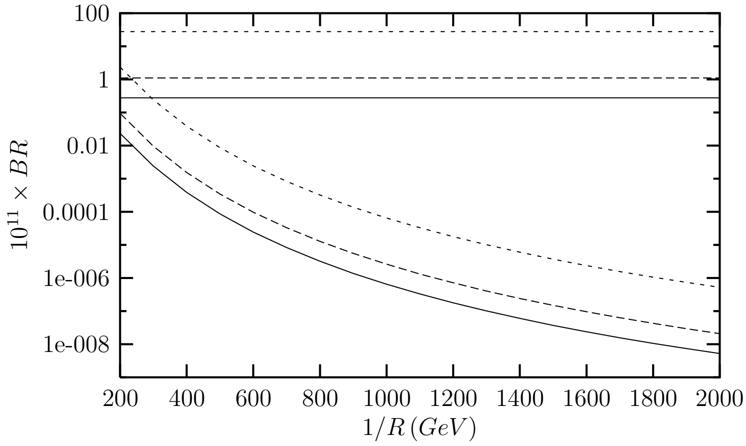

Figure 2: The compactification scale $1/R$ dependence of the BR of the LFV decay $\mu \to e\gamma$ for $m_{h^0} = 100\,\mathrm{GeV}$, $m_{A^0} = 200\,\mathrm{GeV}$, $\bar{\xi}^D_{N,\tau\mu} = 30\,\mathrm{GeV}$, and three different values of the coupling $\bar{\xi}^D_{N,\tau e}$, in the case of one non-universal extra dimension. The solid-dashed-small dashed straight lines (curves) represent the 2HDM (the extra dimension) contribution to the BR for $\bar{\xi}^D_{N,\tau e} = 0.5 \times 10^{-3} - 1.0 \times 10^{-3} - 0.5 \times 10^{-2}\,\mathrm{GeV}$.

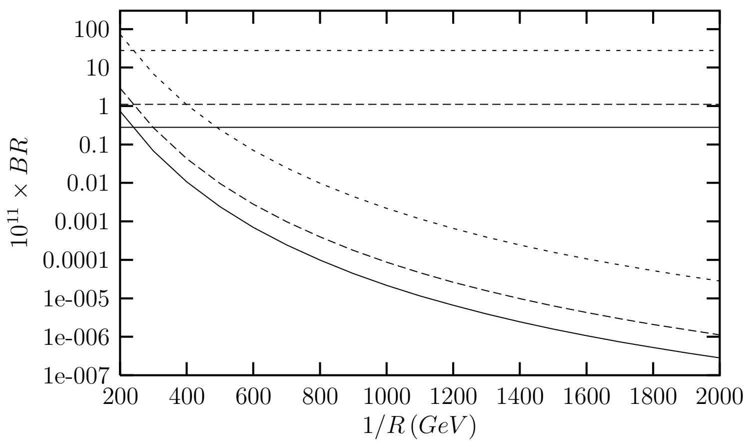

Figure 3: The same as figure 2 but for two non-universal extra dimensions.

to the case of a single extra dimension. On the other hand, the abundance of KK modes in the summation can lead to the divergence in the calculation of the BR. However the the ratio $\frac{m_\tau^2}{m_H^2+m_n^2+m_r^2}$ appearing in the expressions converges to zero sharply with the increasing values of the integers n and r and it leads to the convergence of the KK sum for the compactification scale we study, $1/R > 200\,\mathrm{GeV}$. For the UED case the above ratio has the form $\frac{m_\tau^2+m_n^2+m_r^2}{m_H^2+m_n^2+m_r^2}$ since KK number at each vertex is conserved and KK lepton-lepton-KK neutral Higgs interactions are switched on. Increasing values of n and r forces the ratio to reach one and the convergence problem of the KK sum should be studied carefully.

Figure 2 (3) is devoted to the compactification scale $1/R$ dependence of the BR of the LFV decay $\mu \to e\gamma$, for $m_{h^0} = 100\,\mathrm{GeV}$, $m_{A^0} = 200\,\mathrm{GeV}$, $\bar{\xi}^D_{N,\tau\mu} = 30\,\mathrm{GeV}$, and three different values of the coupling $\bar{\xi}^D_{N,\tau e}$, in the case of one (two) non-universal extra dimension(s). In figure 2, the solid-dashed-small dashed straight lines (curves) represent

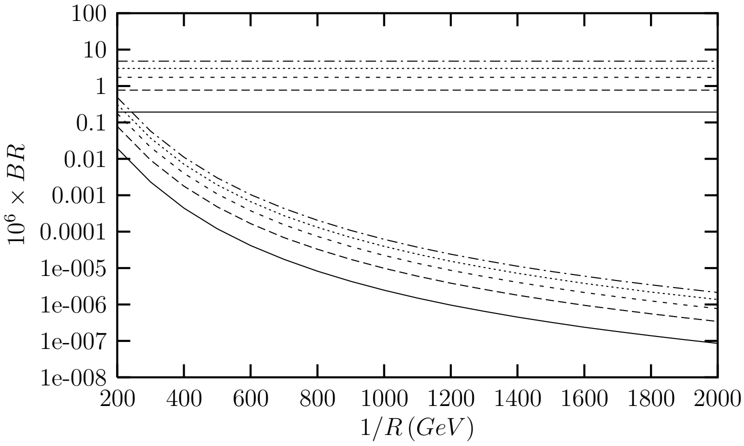

Figure 4: The compactification scale $1/R$ dependence of the BR of the LFV decay $\tau \to \mu\gamma$ for $m_{h^0} = 100\,\mathrm{GeV}$, $m_{A^0} = 200\,\mathrm{GeV}$, $\bar{\xi}^D_{N,\tau\mu} = 30\,\mathrm{GeV}$, and five different values of the coupling $\bar{\xi}^D_{N,\tau\tau}$, in the case of a single non-universal extra dimension. The solid-dashed-small dashed-dotted-dot dashed straight lines (curves) represent the 2HDM (the extra dimension) contribution to the BR for $\bar{\xi}^D_{N,\tau\tau} = 50 - 100 - 150 - 200 - 250\,\mathrm{GeV}$.

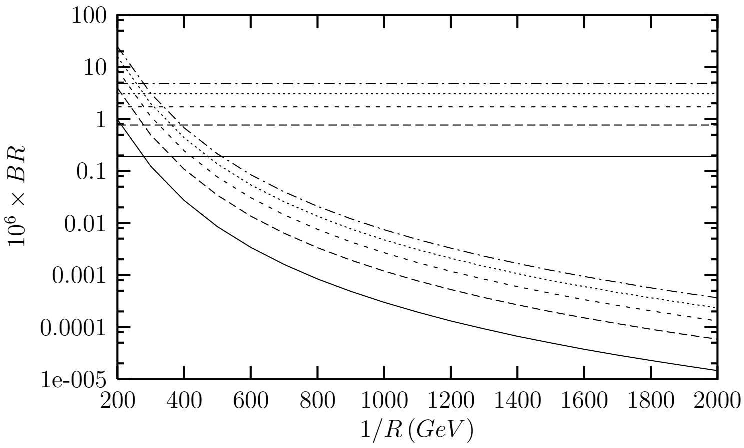

Figure 5: The same as figure 4 but for two non-universal extra dimensions.

the 2HDM (the extra dimension) contribution to the BR for $\bar{\xi}^D_{N,\tau e} = 0.5 \times 10^{-3} - 1.0 \times 10^{-3} - 0.5 \times 10^{-2}\,\mathrm{GeV}$. It is shown that the BR is not sensitive to the extra dimension effects for the single extra dimension. In the case of two non-universal extra dimensions (see figure 3), the sensitivity increases and the effects of extra dimensions can reach even to the main contribution for the small values of compactification scale. This enhancement in the case of two extra dimensions is due to the abundance of KK modes of neutral Higgs bosons.

In figure 4 (5) we present the compactification scale $1/R$ dependence of the BR of the LFV decay $\tau \to \mu\gamma$ for $m_{h^0} = 100\,\mathrm{GeV}$, $m_{A^0} = 200\,\mathrm{GeV}$, $\bar{\xi}^D_{N,\tau\mu} = 30\,\mathrm{GeV}$, and five different values of the coupling $\bar{\xi}^D_{N,\tau\tau}$, in the case of one (two) non-universal extra dimension(s). In figure 4 the solid-dashed-small dashed-dotted-dot dashed straight lines (curves) represent the 2HDM (the extra dimension) contribution to the BR for $\bar{\xi}^D_{N,\tau\tau} =$

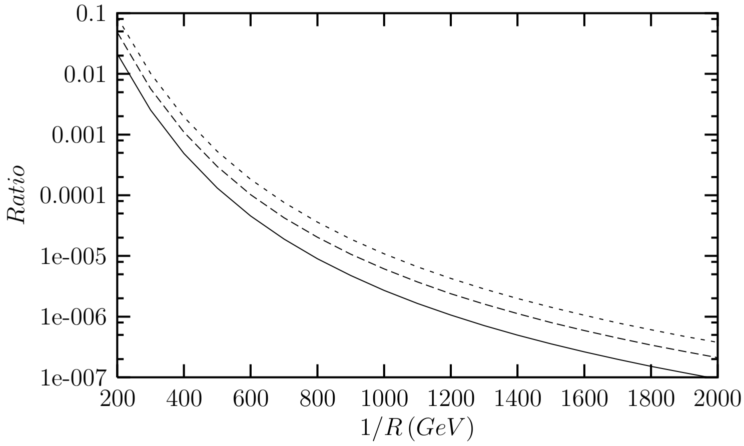

Figure 6: The compactification scale $1/R$ dependence of the *Ratio* for the LFV decay $\tau \to \mu\gamma$ for $m_{h^0} = 100\,\mathrm{GeV}$, $m_{A^0} = 200\,\mathrm{GeV}$, $\bar{\xi}^D_{N,\tau\mu} = 30\,\mathrm{GeV}$, and three different values of the coupling $\bar{\xi}^D_{N,\tau\tau}$, in the case of one non-universal extra dimension. The solid-dashed-small dashed lines represent the *Ratio* for $\bar{\xi}^D_{N,\tau\tau} = 50 - 75 - 100\,\mathrm{GeV}$.

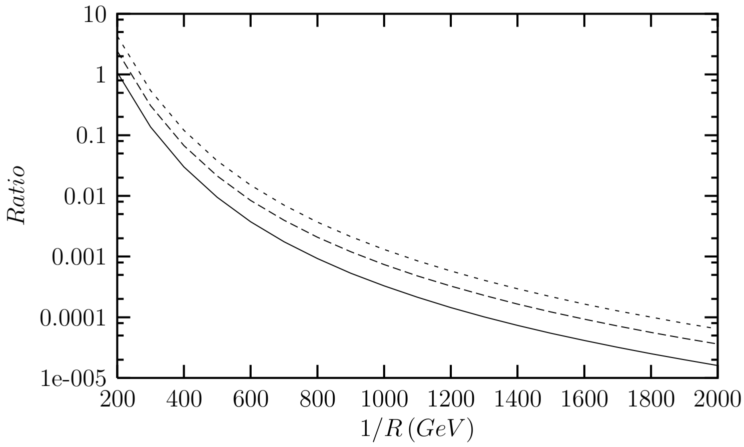

Figure 7: The same as figure 6 but for two non-universal extra dimensions.

$50 - 100 - 150 - 200 - 250\,\mathrm{GeV}$. Similar to the $\mu \to e\gamma$ decay the BR is not sensitive to the extra dimension effects for the single extra dimension. However figure 5 shows that the BR is much more sensitive to the contribution coming from the KK modes in the case of two exta dimensions, in comparison with the single extra one. This sensitivity becomes weak for $1/R \geq 600\,\mathrm{GeV}$.

Now we would like to analyze the ratio $Ratio = \frac{BR^{\mathrm{Ext}}}{BR^{\mathrm{Exp}} - BR^{\mathrm{2HDM}}}$ where BR^{Exp} (BR^{2HDM}) is the experimental upper limit (theoretical value) of the $BR(\tau \to \mu\gamma)$ and BR^{Ext} is the new contributions coming from the extra dimensions.

Figure 6 (7) represents the compactification scale $1/R$ dependence of the *Ratio* for the LFV decay $\tau \to \mu\gamma$ for $m_{h^0} = 100\,\mathrm{GeV}$, $m_{A^0} = 200\,\mathrm{GeV}$, $\bar{\xi}^D_{N,\tau\mu} = 30\,\mathrm{GeV}$, and three different values of the coupling $\bar{\xi}^D_{N,\tau\tau}$ in the case of one (two) non-universal extra dimension(s). In figure 6 (7) the solid-dashed-small dashed lines represent the *Ratio* for

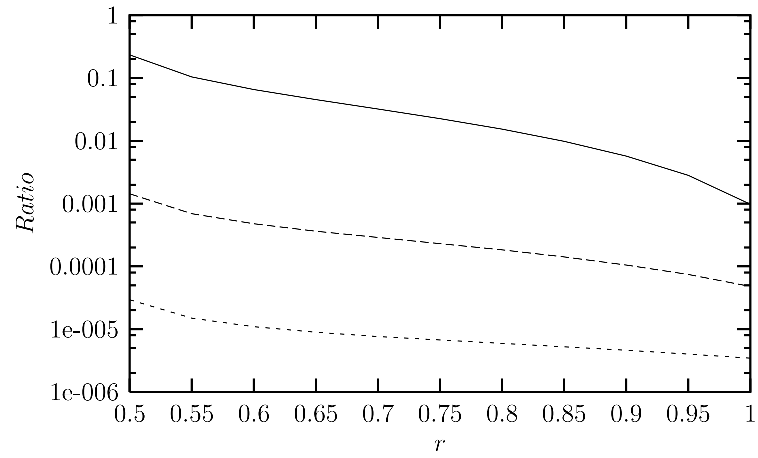

Figure 8: The parameter $r = \frac{m_h^0}{m_A^0}$ dependence of the *Ratio* for the $\tau \to \mu\gamma$ for $m_{A^0} = 200\,\mathrm{GeV}$, $\bar{\xi}^D_{N,\tau\mu} = 30\,\mathrm{GeV}$, $\bar{\xi}^D_{N,\tau\tau} = 100\,\mathrm{GeV}$, for three different values of the compactification scale $1/R$, $1/R = 200, 500, 1000\,\mathrm{GeV}$ in the case of one non-universal extra dimension.

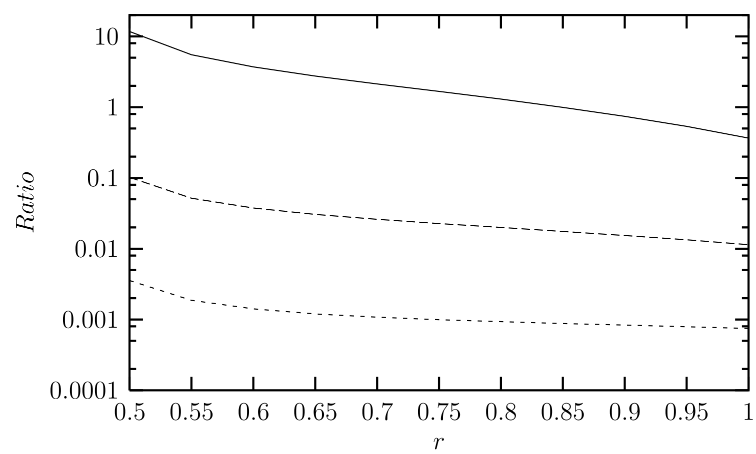

Figure 9: The same as figure 8 but for two non-universal extra dimensions.

$\bar{\xi}^D_{N,\tau\tau} = 50 - 75 - 100\,\mathrm{GeV}$, for a single (double) non-universal extra dimension. Figure 6 shows that the discrepancy between the experimental value and the 2HDM prediction cannot be compensated with the addition of extra dimensions for the given values of the Yukawa coupling, $\bar{\xi}^D_{N,\tau\tau} = 50-75-100\,\mathrm{GeV}$. On the other hand this ratio increases with the increasing values of the coupling constant $\bar{\xi}^D_{N,\tau\tau}$. For two non-universal extra dimensions it is possible to cover the discrepancy between the experimental and the theoretical values for the small values of the compactification scale $1/R$. For $600 \leq 1/R \leq 800\,\mathrm{GeV}$ the magnitude of extra dimension effect becomes almost 1% of the difference of the experimental and the theoretical values of the BR.

In figure 8 (9) we present the parameter $r = \frac{m_{h^0}}{m_{A^0}}$ dependence of the *Ratio* for the $\tau \to \mu\gamma$ decay for $m_{A^0} = 200\,\mathrm{GeV}$, $\bar{\xi}^D_{N,\tau\mu} = 30\,\mathrm{GeV}$, $\bar{\xi}^D_{N,\tau\tau} = 100\,\mathrm{GeV}$, and three different values of the compactification scale $1/R$, $1/R = 200, 500, 1000\,\mathrm{GeV}$ in the case of one (two) non-universal extra dimension(s). These figures show that the *Ratio* is sensitive to the mass ratio of neutral Higgs bosons h^0 and A^0 especially for small $1/R$. The increasing values of r causes to decrease the ratio and this sensitivity becomes weak for the large values of the scale $1/R$. In the case that the neutral Higgs masses are nearly degenerate the *Ratio* is suppressed almost one order compared to the case that the parameter $r = 0.5$ for the intermediate values of $1/R$.

Now we would like to present the results briefly.

- The single non-universal extra dimension effects on the BR's of $\mu \to e\gamma$ and $\tau \to \mu\gamma$ decays are considerably weak, however, the addition of one more spatial dimension causes relatively strong sensitivity on the BR's.

- The contribution coming from the single non-universal extra dimension is not enough large to compensate the difference between the experimental and the theoretical values of the BR, for the values of the Yukawa coupling $\bar{\xi}^D_{N,\tau\tau} < 100\,\mathrm{GeV}$. In the case of two non-universal extra dimensions this discrepancy can be covered by KK modes of neutral Higgs bosons for the small values of the compactification scale $1/R$.

- The extra dimension contributions are sensitive to the mass ratio of neutral Higgs bosons h^0 and A^0 and it becomes weaker with the increasing values of the compactification scale $1/R$.

As a final comment, the effect of the single non-universal extra dimension on the BR's of LFV decays $\mu \to e\gamma$ and $\tau \to \mu\gamma$ is weak. However, their contributions enhance in the case of two extra dimensions and the more accurate future experimental results of these decays, hopefully, will be helpful to analyze the possible signals coming from the extra dimensions.

Acknowledgments

This work has been supported by the Turkish Academy of Sciences in the framework of the Young Scientist Award Program. (EOI-TUBA-GEBIP/2001-1-8)

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JHEP08(2004)020